

# Programme de la journée SIGMA - MODE du 30 janvier 2024.

## Accueil des participants : 9h15 - café

- 9h30 - 10h20 : Julie Delon (Université Paris Cité)
- 10h20 - 10h50 : Alex Delalande (Lagrange Center)
- Pause - café - 11h15 - 11h45 : Flavien Léger (INRIA)
- 11h45 - 12h15 : Edouard Pauwels (Toulouse Capitole University)
- Repas du midi -
- 14h - 14h50 : Nicolas Boumal (EPFL)
- 14h50 - 15h20 : Julie Digne (CNRS)
- Pause- café
- 15h50 - 16h40 : Vincent Duval (INRIA)
- 16h40 - 17h10 : Anna Korba (ENSAE/CREST)

## Titres et résumés provisoires:

### Nicolas Boumal:

#### Nonconvex just means not convex

A nonconvex optimization problem is just one that happens not to be convex. Whether that's bad or not depends on your aims and computational budget. Some problems turn out to have a fairly benign nonconvex landscape, or one that can be efficiently navigated by particular algorithms. It is useful to get a better understanding of such problems: What are they? How can we recognize them? Which algorithms are suitable for them? I will touch on aspects of those questions that we research in my group.

### Alex Delalande:

#### Quantitative Stability of the Pushforward Operation by an Optimal Transport Map

We study the quantitative stability of the mapping that to a measure associates its pushforward measure by a fixed (non-smooth) optimal transport map. We exhibit a tight Hölder-behavior for this operation under minimal assumptions. Our proof essentially relies on a new bound that quantifies the size of the singular sets of a convex and Lipschitz continuous function on a bounded domain.

## **Julie Delon:**

### **Optimal transport with invariances between Gaussian mixture models**

Gaussian Mixture Models (GMMs) are ubiquitous in statistics and machine learning and are especially useful in applied fields to represent probability distributions of real datasets. Optimal transport can be used to compute distances or geodesics between such mixture models, but the corresponding Wasserstein geodesics do not preserve the property of being a GMM. It has been shown in <https://arxiv.org/abs/1907.05254> that restricting the set of possible coupling measures to GMMs transforms the original infinitely dimensional optimal transport problem into a finite dimensional problem with a simple discrete formulation, well suited to applications where a clustering structure is present in the data.

In this talk, we present two possible extensions of this Wasserstein-type distance between GMMs that remain invariant to isometries. Inspired by the Gromov-Wasserstein distance, these extensions can also be used to compare GMMs of different dimensions.

## **Vincent Duval:**

### **Recovery of piecewise constant images using total (gradient) variation regularization**

Total variation regularization has been widely used in inverse problems arising in image processing, following the work of Rudin, Osher and Fatemi. The conventional wisdom is that this regularization is well suited to the recovery of piecewise constant images. In this talk, I will describe how the recovery of such images is related to geometric variational problems. By studying their stability properties, it is possible to derive a condition which ensures that, at low noise, the reconstructed solutions have exactly the same number of constant components, which converge both in shape and amplitudes towards those of the unknown.

In a second part I will describe a derive an "off-the-grid" discretization scheme and optimization algorithm which exploit these properties by relying on the Frank-Wolfe algorithm.

It is a joint work with Romain Petit and Yohann De Castro.

**Anna Korba:**

### **Sampling through optimization of discrepancies**

Sampling from a target measure when only partial information is available (e.g. unnormalized density as in Bayesian inference, or true samples as in generative modeling )

is a fundamental problem in computational statistics and machine learning. The sampling problem can be formulated as an optimization over the space of probability distributions of a well-chosen discrepancy (e.g. a divergence or distance).

In this talk, we'll discuss several properties of sampling algorithms for some choices of discrepancies (well-known ones, or novel proxies), both regarding their optimization and quantization aspects.

**Flavien Léger:**

### **An intrinsic geometry for alternating minimization**

Convergence rates for the alternating minimization method (AM) can be obtained under a half-forgotten condition introduced by Csiszár and Tusnády in 1984, the five-point property (FPP).

When the dimensions of the two underlying spaces are equal, we can attach to AM an intrinsic pseudo-Riemannian geometry introduced by Kim and McCann in the field of optimal transport.

Under nonnegative or nonpositive curvature, we will show that the FPP can be understood as a certain convexity of the problem on specific geodesics for the Kim-McCann geometry.

This allows us to recover various convergence rates under a unified perspective as well as to obtain new rates.

Joint work with Pierre-Cyril Aubin.

**Édouard Pauwels:**

### **Nonsmooth differentiation of parametric fixed points**

Recent developments in the practice of numerical programming require optimization problems not only to be solved numerically, but also to be differentiated. This allows to integrate the computational operation of evaluating a solution in larger models, which are themselves trained or optimized using gradient methods. Most well known applications include bi-level optimization and implicit input-output relations in deep network models. Fixed point of contraction mappings provide a natural high level description of

these applications. We will briefly describe the interplay between implicit differentiation and derivatives of fixed point iterations in the smooth settings, dating back to early works in automatic differentiation. Motivated by the possibility to differentiate solutions of nonsmooth or constrained problems, we will describe recent extension of these results to the conservative gradient setting, a notion of generalized derivative which is compatible with calculus and gradient type optimization methods. We will also cover explicit examples related to the differentiation of solutions to monotone inclusions.

Joint work with Jérôme Bolte, Tony Silveti-Falls, Tam Le, Samuel Vaïter.