About the use of weak transport costs for concentration and functionals inequalities.

Based on :
- Kantorovich duality for general transport costs and applications.
J. Funct. Anal. 273 (2017), no. 11, 3327-3405.
Joint work with N. Gozlan, C. Roberto et P. Tetali.

Projet Bézout : New challenging Monge problems and their applications

Avril 2022

P-M. Samson
Université Gustave Eiffel

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- $H(\nu|m)$: the relative entropy of $\nu \in \mathcal{P}(\mathcal{X})$ with respect to a measure m,

$$H(\nu|m) := \int \log\left(\frac{d\nu}{dm}\right) d\nu, \qquad \text{if } \nu << m,$$

  and $H(\nu|m) := +\infty$ otherwise.

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In the 1990s, K. Marton introduced a weak transport cost $\widetilde{T}_2(\nu|\mu)$.
She proved a variant of the Csizàr-Kullback-Pinsker inequality to recover a Talagrand's concentration inequality on product spaces, related to the so-called convex-hull method.

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The Csizár-Kullback-Pinsker inequality :

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The Csizár-Kullback-Pinsker inequality : for any $\mu, \nu \in \mathcal{P}(\mathcal{X})$

$$\|\mu - \nu\|_{TV}{}^2 \leqslant 2\, H(\nu|\mu),$$

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Introduction : Marton's transport inequality

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She proved a variant of the Csizàr-Kullback-Pinsker inequality to recover a Talagrand's concentration inequality on product spaces, related to the so-called convex-hull method.

The Csizár-Kullback-Pinsker inequality : for any $\mu, \nu \in \mathcal{P}(\mathcal{X})$

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By Jensen's inequality,

$$\frac{1}{4}\|\mu - \nu\|_{TV}^2 \leqslant \widetilde{T}_2(\nu|\mu) \leqslant \frac{1}{2}\|\mu - \nu\|_{TV}$$

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p_i denotes the i-th marginal of p.

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$$\frac{1}{2}\widetilde{T}_2(\nu_2|\nu_1) \leqslant \left(\sqrt{H(\nu_1|\mu)} + \sqrt{H(\nu_2|\mu)}\right)^2, \qquad \forall \mu \in \mathcal{P}(\mathcal{X}), \nu_1, \nu_2 \in \mathcal{P}(\mathcal{X}),$$

or equivalently, since $\left(\sqrt{H_1} + \sqrt{H_2}\right)^2 = \inf_{s\in(0,1)} \{H_1/s + H_2/(1-s)\}$,

$$\frac{1}{2}\widetilde{T}_2(\nu_2|\nu_1) \leqslant \frac{1}{s}H(\nu_1|\mu) + \frac{1}{1-s}H(\nu_2|\mu), \qquad \forall s \in (0,1).$$

Transport-entropy inequalities tensorize : setting $\mu^n = \mu \times \cdots \times \mu \in \mathcal{P}(\mathcal{X}^n)$,

$$\frac{1}{2}\widetilde{T}_2(\nu_2|\nu_1) \leqslant \frac{1}{s}H(\nu_1|\mu^n) + \frac{1}{1-s}H(\nu_2|\mu^n), \qquad \forall s \in (0,1), \quad \forall \nu_1, \nu_2 \in \mathcal{P}(\mathcal{X}^n).$$

where

$$\widetilde{T}_2(\nu_2|\nu_1) := \inf_{\substack{\pi \in \Pi(\nu_1,\nu_2) \\ \pi = \nu_1 \oslash p}} \int c^n(x, p_x)\, d\nu_1(x),$$

with for $x = (x_1, \ldots, x_n) \in \mathcal{X}^n$,

$$c^n(x,p) := \sum_{i=1}^n c(x_i, p_i), \qquad c(x_i,p_i) = \left(\int \mathbb{1}_{x_i \neq y_i}\, dp_i(y_i)\right)^2.$$

p_i denotes the i-th marginal of p.

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The function F is convex in p

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Links with Talagrand's convex-hull distance :

$$D_{Tal}(x,A) = \sup_{\alpha\in B_1}\inf_{y\in A}\sum_{i=1}^n \alpha_i 1_{x_i\neq y_i} \qquad B_1 \text{ : the Euclidean ball in } \mathbb{R}^n.$$

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(F is linear in p, the infimum is reached at the extremal points of $\mathcal{P}(A)$.)

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$$c: \begin{array}{ccc} \mathcal{X} \times \mathcal{P}_\gamma(\mathcal{X}) & \to & [0,+\infty] \\ (x,p) & \mapsto & c(x,p), \end{array}$$

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Extension of Kantorovich duality to weak transport cost

Definition : Weak optimal transport cost

Let us consider a measurable function

$$c : \begin{array}{ccc} \mathcal{X} \times \mathcal{P}_\gamma(\mathcal{X}) & \to & [0, +\infty] \\ (x, p) & \mapsto & c(x, p), \end{array}$$

The weak optimal cost, $\mathcal{T}_c(\nu|\mu)$, associated to c is defined by

$$\mathcal{T}_c(\nu|\mu) := \inf_{\substack{\pi \in \Pi(\mu,\nu) \\ \pi = \mu \oslash p}} \int c(x, p_x) d\mu(x), \qquad \mu, \nu \in \mathcal{P}_\gamma(\mathcal{X}),$$

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For $\gamma_0(d(x, y)) = \mathbb{1}_{x \neq y}$ and $\alpha(h) = h^2$,

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Kantorovich duality for weak transport costs

$\Phi_\gamma(\mathcal{X})$: the set of continuous functions $\varphi : \mathcal{X} \to \mathbb{R}$ such that

$$|\varphi(x)| \leqslant a + b\,\gamma(d(x, x_0)), \qquad \forall x \in \mathcal{X}.$$

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where for $\varphi \in \Phi_{\gamma,b}(\mathcal{X})$,

$$R_c\varphi(x) = \inf_{p \in \mathcal{P}_\gamma(\mathcal{X})} \left\{ \int \varphi\, dp + c(x, p) \right\}, \qquad x \in \mathcal{X}.$$

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Kantorovich duality for weak transport costs

$\Phi_\gamma(\mathcal{X})$: the set of continuous functions $\varphi : \mathcal{X} \to \mathbb{R}$ such that

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Definition : duality for weak transport costs

One says that duality holds for the cost

$$c : \mathcal{X} \times \mathcal{P}_\gamma(\mathcal{X}) \to [0, +\infty],$$

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Main assumptions for duality to hold :
- $p \mapsto c(x, p)$ is convex, - semi-continuity assumptions.

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Theorem [GRST ’15] : Examples of weak costs for which duality holds

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Theorem [GRST '15] : Examples of weak costs for which duality holds

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Theorem [GRST '15] : Examples of weak costs for which duality holds

Example 2 : Let μ_0 denotes a reference probability measure on $\mathcal{X}$.

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Theorem [GRST '15] : Examples of weak costs for which duality holds

Example 2 : Let μ_0 denotes a reference probability measure on $\mathcal{X}$.

$$c(x,p) = \int \beta\left(\gamma(d(x,y))\frac{dp}{d\mu_0}(y)\right) d\mu_0(y), \qquad \text{if } p << \mu_0,$$

and $c(x,p) = +\infty$ otherwise, with $\beta : \mathbb{R}^+ \to [0,+\infty]$, convex and $\beta(0) = 0$.

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$$\begin{aligned}\widehat{\mathcal{T}}_\beta(\nu|\mu) &= \inf_{\pi\in\Pi(\mu,\nu)} \iint \beta\left(\gamma(d(x,y))\frac{dp_x}{d\mu_0}(y)\right) d\mu_0(y)d\mu(x)\\ &\geqslant \widetilde{\mathcal{T}}_\beta(\nu|\mu)\end{aligned}$$

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$$\widehat{\mathcal{T}}_\beta(\nu|\mu) = \sup_\varphi \left\{ \int \widehat{Q}_\beta\varphi(x)\, d\mu(x) - \int \varphi(y)\, d\nu(y) \right\},$$

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$$c(x,p) = \int \beta\left(\gamma(d(x,y))\frac{dp}{d\mu_0}(y)\right) d\mu_0(y), \qquad \text{if } p << \mu_0,$$

and $c(x,p) = +\infty$ otherwise, with $\beta : \mathbb{R}^+ \to [0,+\infty]$, convex and $\beta(0) = 0$.

$$\begin{aligned}\widehat{\mathcal{T}}_\beta(\nu|\mu) &= \inf_{\pi\in\Pi(\mu,\nu)} \iint \beta\left(\gamma(d(x,y))\frac{dp_x}{d\mu_0}(y)\right) d\mu_0(y)d\mu(x)\\ &\geqslant \widetilde{\mathcal{T}}_\beta(\nu|\mu)\end{aligned}$$

$$\widehat{\mathcal{T}}_\beta(\nu|\mu) = \sup_\varphi \left\{\int \widehat{Q}_\beta\varphi(x)\, d\mu(x) - \int \varphi(y)\, d\nu(y)\right\},$$

$$\widehat{Q}_\beta\varphi(x) = \inf_{p\in\mathcal{P}_\gamma(X)} \left\{\int \varphi(y)\, dp(y) + \int \beta\left(\gamma(d(x,y))\frac{dp}{d\mu_0}(y)\right) d\mu_0(y)\right\}.$$

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Particular case : a Talagrand's cost for $\gamma_0(u) = \mathbb{1}_{u\neq 0}$,

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used by Talagrand (1996) as a main ingredient to reach deviation inequalities for supremum of empirical processes with Bernstein's bounds, see also S. (2007).

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Remark : This cost has strong connections with convex functions.

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The function $\overline{\varphi}$ is convex.

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where the supremum runs over all convex Lipschitz functions $\overline{\varphi} : \mathbb{R}^m \to \mathbb{R}$ bounded from below,

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where the supremum runs over all convex Lipschitz functions $\overline{\varphi} : \mathbb{R}^m \to \mathbb{R}$ bounded from below, and $Q_\theta\overline{\varphi}$ is the usual infimum-convolution operator.

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A first use of barycentric cost for a Strassen result

$$\overline{\mathcal{T}}_\theta(\nu|\mu) = \sup_{\psi \text{ convex}} \left\{ \int Q_\theta \psi \, d\mu - \int \psi \, d\nu \right\},$$

with

$$Q_\theta \psi(x) = \inf_{z \in \mathbb{R}^m} \{\psi(z) + \theta(x - z)\}, \quad x \in \mathbb{R}^m$$

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$$\overline{\mathcal{T}}_\theta(\nu|\mu) = \sup_{\psi \ \text{convex}} \left\{ \int Q_\theta \psi \, d\mu - \int \psi \, d\nu \right\},$$

with

$$Q_\theta \psi(x) = \inf_{z \in \mathbb{R}^m} \{\psi(z) + \theta(x - z)\}, \quad x \in \mathbb{R}^m$$

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Proposition. [GRST 2015]

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Let $\mu, \nu \in \mathcal{P}_1(\mathbb{R}^m)$; one says that μ is dominated by ν in the convex order sense, $\mu \preceq_C \nu$, if

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Theorem. [Strassen 1965]

Let $\mu, \nu \in \mathcal{P}(\mathbb{R}^m)$.Then $\mu \preceq_C \nu$ if and only if there exists a martingale (X, Y) ($\mathbb{E}[Y|X] = X$), where X follows the law μ and Y the law ν.

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Further results for barycentric costs

For $\theta(x-z)=|x-z|^2$,

$$\overline{\mathcal{T}}_\theta(\nu|\mu)=\overline{\mathcal{T}}_2(\nu|\mu)=\inf_{(X,Y)}\mathbb{E}\left[\left|X-\mathbb{E}[Y|X]\right|^2\right].$$

$$W_2^2(\nu,\mu)=\inf_{(X,Y)}\mathbb{E}\left[|X-Y|^2\right].$$

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For fixed $\nu\in\mathcal{P}_2(\mathbb{R}^m)$ let

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Proposition : Gozlan-Juillet 2018, Alfonsi-Corbetta-Jourdain 2018

Given $\mu,\nu \in \mathcal{P}_2(\mathbb{R}^m)$, there exists a unique probability measure $\overline{\mu} \in \mathcal{B}_\nu$ such that

$$\overline{\mathcal{T}}_2(\nu|\mu) = W_2^2(\overline{\mu},\nu) = \inf_{\eta\in\mathcal{B}_\nu} W_2^2(\eta,\nu).$$

$\overline{\mu}$ is called projection of μ on $\mathcal{B}_\nu$.

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$\overline{\mu}$ is called projection of μ on $\mathcal{B}_\nu$.

Theorem : (equality case) Gozlan-Juillet 2018

$\mu,\nu \in \mathcal{P}_2(\mathbb{R}^m)$. With the above definitions,

$$\overline{\mathcal{T}}_2(\nu|\mu) = W_2^2(\mu,\nu) \quad \Leftrightarrow \quad \overline{\mu} = \mu \quad \Leftrightarrow \quad \nu = \nabla\tau\#\mu,$$

where $\tau : \mathbb{R}^m \to \mathbb{R}$ is a convex function of class $\mathcal{C}_1$ such that $\nabla\tau$ is 1-Lipschitz.

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A complete statement for $\overline{T}_2$

Theorem : Gozlan-Juillet 2018, Backhoff-Veraguas-Beiglböck-Pammer 2018

- There exists $\psi^o : \mathbb{R}^m \to \mathbb{R}$ convex bounded from below such that

$$\overline{\mathcal{T}}_2(\nu|\mu) = \int Q_2\psi^o \, d\mu - \int \psi^o \, d\nu.$$

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- $\overline{\mu} = \nabla\tau^o \# \mu$ where

$$\tau^o = h^\star \quad \text{with} \quad h(x) := \frac{\psi^o(x) + |x|^2}{2}, \quad x \in \mathbb{R}^m.$$

(recall that $h^\star(y) = \sup_{x\in\mathbb{R}^m}\{x.y - h(y)\}$.)
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(recall that $h^\star(y) = \sup_{x \in \mathbb{R}^m}\{x.y - h(y)\}$.)
$\nabla\tau^o$ is 1-Lipschitz.

- If (X, Y) is a coupling of μ and ν such that

$$\overline{\mathcal{T}}_2(\nu|\mu) = \mathbb{E}\left[|X - \mathbb{E}[Y|X]|^2\right],$$

then $\mathbb{E}[Y|X]$ has law $\overline{\mu}$ and $\mathbb{E}[Y|X] = \nabla\tau^o(X)$ almost surely.

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Examples of weak optimal transport costs for which duality holds

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Examples of weak optimal transport costs for which duality holds

Example 4 : The martingale transport problem on the line.

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Examples of weak optimal transport costs for which duality holds

Example 4 : The martingale transport problem on the line.
Let $\mu, \nu \in \mathcal{P}(\mathbb{R})$ such that $\mu \preceq_C \nu$. According to Strassen Theorem,

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The dual Kantorovich Theorem for weak cost applies

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The dual Kantorovich Theorem for weak cost applies and we recover the duality result by Beighböck-Henry-Labordère-Penker (2013).

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Theorem : [B and al.,2013]

Let $w : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ be a upper semi-continuous function, bounded from above.

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Duality for martingale costs

Theorem : [B and al.,2013]

Let $w : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ be a upper semi-continuous function, bounded from above.

$$\sup_{\pi \in \Pi^{mart}(\mu,\nu)} \iint w \, d\pi = \inf_{f,g,\gamma} \left\{ \int f \, d\mu + \int g \, d\nu \right\},$$

where the infimum runs over all measurable bounded functions f, g, γ such that for all $x, y \in \mathbb{R}$, $\quad w(x, y) \leqslant f(x) + g(y) + \gamma(x)(y - x)$.

idea of the proof : The inequality $\leqslant$ is obvious since for all $\pi \in \Pi^{mart}(\mu, \nu)$, $\int h(x)(y - x) \, d\pi(x, y) = 0$. For the reverse inequality $\geqslant$: let $\varepsilon > 0$, $\omega = -w$.

$$\sup_{\pi \in \Pi^{mart}(\mu,\nu)} \iint w \, d\pi \geqslant \int (-R_c g_0) \, d\mu + \int g_0 \, d\nu - \varepsilon.$$

Observe that $\quad i(x, p) = \sup_{\gamma \in \mathbb{R}} \gamma \cdot \left(\int y \, dp(y) - x \right),$

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over all f_0, g_0, γ, $\quad f_0(x) + g_0(y) + \gamma(x) \cdot (y-x) + \varepsilon \geqslant w(x,y)$.

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Applications of duality to transport-entropy inequalities and concentration

A well known example : Pinsker $\Leftrightarrow$ Inégalité exponentielle (Hoeffding).

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Definition : Weak transport-entropy inequality $\mathbf{T}_c(a_1, a_2)$

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Proposition : Dual characterization for weak transport-entropy inequalities.

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Proposition : Dual characterization for weak transport-entropy inequalities.

If the Kantorovich duality holds for the weak cost $\mathcal{T}_c$, then the following statements are equivalents :

i) μ satisfies $\mathbf{T}_c(a_1, a_2)$ $(a_1, a_2 > 0)$

ii) For all functions $\varphi \in \Phi_{\gamma,b}(\mathcal{X})$,

$$\left(\int e^{\frac{R_c\varphi}{a_2}}\, d\mu\right)^{a_2} \left(\int e^{-\frac{\varphi}{a_1}}\, d\mu\right)^{a_1} \leqslant 1$$

$$R_c\varphi(x) = \inf_{p \in \mathcal{P}_\gamma(\mathcal{X})} \left\{ \int \varphi(y)\, dp(y) + c(x,p) \right\}, \qquad x \in \mathcal{X}.$$

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ii) is a generalisation of the so-called (convex) τ-property introduced by Maurey (1990) to recover Talagrand's concentration results.

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Assume that for all $\nu_1, \nu_2 \in \mathcal{P}_\gamma(\mathcal{X})$,

$$\mathcal{T}_c(\nu_1|\nu_2) \leqslant a_1 H(\nu_1|\mu) + a_2 H(\nu_2|\mu)$$

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$$a_2 \left(\int \frac{R_c\varphi}{a_2} \, d\nu_2 - H(\nu_2|\mu) \right) + a_1 \left(\int -\frac{\varphi}{a_1} \, d\nu_1 - H(\nu_1|\mu) \right) \leqslant 0.$$

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By optimizing over all ν_1, ν_2 we get

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Therefore, for all $\varphi \in \Phi_{\gamma,b}(X)$,

$$a_2 \sup_{\nu_2} \left\{ \int \frac{R_c\varphi}{a_2} \, d\nu_2 - H(\nu_2|\mu) \right\} + a_1 \sup_{\nu_1} \left\{ \int -\frac{\varphi}{a_1} \, d\nu_1 - H(\nu_1|\mu) \right\} \leqslant 0.$$

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Since $\displaystyle \sup_{\nu \in \mathcal{P}_\gamma(\mathcal{X})} \left\{ \int \psi \, d\nu - H(\nu|\mu) \right\} = \log \int e^{\psi} \, d\mu, \quad \forall \psi \in \Phi_{\gamma,b}(X),$

Idea of the proof (Bobkov-Götze 1999)

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Therefore, for all $\varphi \in \Phi_{\gamma,b}(X)$,

$$a_2 \sup_{\nu_2} \left\{ \int \frac{R_c\varphi}{a_2} \, d\nu_2 - H(\nu_2|\mu) \right\} + a_1 \sup_{\nu_1} \left\{ \int -\frac{\varphi}{a_1} \, d\nu_1 - H(\nu_1|\mu) \right\} \leqslant 0.$$

Since $\sup_{\nu \in \mathcal{P}_\gamma(\mathcal{X})} \left\{ \int \psi \, d\nu - H(\nu|\mu) \right\} = \log \int e^{\psi} \, d\mu, \quad \forall \psi \in \Phi_{\gamma,b}(X),$

it follows that

$$a_2 \log \int e^{R_c\varphi/a_2} \, d\mu + a_1 \log \int e^{-\varphi/a_1} \, d\mu \leqslant 0.$$

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Therefore, for all $\varphi \in \Phi_{\gamma,b}(X)$,

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Since $\sup_{\nu \in \mathcal{P}_\gamma(\mathcal{X})} \left\{ \int \psi \, d\nu - H(\nu|\mu) \right\} = \log \int e^{\psi} \, d\mu, \quad \forall \psi \in \Phi_{\gamma,b}(X),$

it follows that

$$a_2 \log \int e^{R_c\varphi/a_2} \, d\mu + a_1 \log \int e^{-\varphi/a_1} \, d\mu \leqslant 0.$$

or equivalently

$$\left(\int e^{R_c\varphi/a_2} \, d\mu \right)^{a_2} \left(\int e^{-\varphi/a_1} \, d\mu \right)^{a_1} \leqslant 1$$

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From dual characterization of transport-entropy inequality to concentration

We assume that for all measurable functions $\varphi : \mathcal{X} \to \mathbb{R} \cup \{+\infty\}$ bounded from below

$$\left(\int e^{\frac{R_c\varphi}{a_2}}\,d\mu\right)^{a_2}\left(\int e^{-\frac{\varphi}{a_1}}\,d\mu\right)^{a_1} \leqslant 1$$

where $\quad R_c\varphi(x) = \inf_{p\in\mathcal{P}_\gamma(\mathcal{X})}\left\{\int \varphi\,dp + c(x,p)\right\}, \qquad x \in \mathcal{X}.$

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Let $A \subset \mathcal{X}$.

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Let $A \subset \mathcal{X}$. Applying this inequality to the function

$$\varphi(x) = i_A(x) := \begin{cases} 0 \text{ if } x \in A, \\ +\infty \text{ otherwise}, \end{cases}$$

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Let $A \subset \mathcal{X}$. Applying this inequality to the function

$$\varphi(x) = i_A(x) := \begin{cases} 0 \text{ if } x \in A, \\ +\infty \text{ otherwise,} \end{cases}$$

since $\int e^{-\frac{i_A}{a_1}}\, d\mu = \mu(A)$,

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and $R_c i_A(x) = \inf_{p\in\mathcal{P}(\mathcal{X})} \left\{ \int i_A dp + c(x,p) \right\} = \inf_{p, p(A)=1} c(x,p)$

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Let $A \subset \mathcal{X}$. Applying this inequality to the function

$$\varphi(x) = i_A(x) := \begin{cases} 0 \text{ if } x \in A, \\ +\infty \text{ otherwise,} \end{cases}$$

since $\int e^{-\frac{i_A}{a_1}}\,d\mu = \mu(A)$,

and $R_c i_A(x) = \inf_{p\in\mathcal{P}(\mathcal{X})}\left\{\int i_A dp + c(x,p)\right\} = \inf_{p,p(A)=1} c(x,p) := c(x,A)$,

we get the following type of Talagrand's concentration result

$$\left(\int e^{\frac{c(x,A)}{a_2}}\,d\mu(x)\right)^{a_2}\mu(A)^{a_1} \leqslant 1.$$

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where $\quad R_c\varphi(x) = \inf_{p\in\mathcal{P}_\gamma(\mathcal{X})}\left\{\int \varphi\,dp + c(x,p)\right\}, \qquad x \in \mathcal{X}.$

Let $A \subset \mathcal{X}$. Applying this inequality to the function

$$\varphi(x) = i_A(x) := \begin{cases} 0 \text{ if } x \in A, \\ +\infty \text{ otherwise,} \end{cases}$$

since $\int e^{-\frac{i_A}{a_1}}\,d\mu = \mu(A)$,

and $R_c i_A(x) = \inf_{p\in\mathcal{P}(\mathcal{X})}\left\{\int i_A dp + c(x,p)\right\} = \inf_{p,p(A)=1} c(x,p) := c(x,A)$,

we get the following type of Talagrand's concentration result

$$\left(\int e^{\frac{c(x,A)}{a_2}}\,d\mu(x)\right)^{a_2}\mu(A)^{a_1} \leqslant 1.$$

By Markov inequality,

$$\mu(\mathcal{X}\backslash A_t) = \mu\big(\{x \in \mathcal{X}, c(x,A) > t\}\big)$$

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From dual characterization of transport-entropy inequality to concentration

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Proposition [S. 2003] : Weak transport inequalities for the Bernoulli measure

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Proposition [GRST 2015] : Weak transport inequalities for the Poisson measure

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Proposition [GRST 2015] : Weak transport inequalities for the Poisson measure

Choose $q = \lambda/n$, $\lambda > 0$, and use the weak convergence as $n \to +\infty$ of the binomial law $\mu_{\lambda/n,n}$ to the Poisson measure $p_\lambda(k) = \frac{\lambda^k}{k!}e^{-\lambda}$, $k \in \mathbb{N}$.

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Let $\theta : \mathbb{R} \to \mathbb{R}^+$ be a symmetric convex cost function satisfying

$$\theta(t) = t^2, \qquad \forall t \leqslant t_o, \quad \text{for some } t_o > 0.$$

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For any $\mu, \nu \in \mathcal{P}(\mathbb{R})$, we consider the barycentric transport cost

$$\overline{\mathcal{T}}_{\theta_a}(\nu|\mu) = \inf_{\pi \in \Pi(\mu,\nu)} \int \theta_a \left(\int x - \int y \, dp_x(y) \right) d\mu(x).$$

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Theorem : [Gozlan-Roberto-S.-Shu-Tetali 2017]

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Used by Strzelecka-Strzelecki-Tkocz (2017) to show that any symmetric probability measure with log-concave tails satisfies a barycentric transport inequality with optimal cost, up to a universal constant.

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$A_{c,r} \subset A_{\sqrt{nr}}$,

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$A_{c,r} \subset A_{\sqrt{nr}}$, setting $t = \sqrt{nr}$ we recover Maurey's concentration inequality.

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  Transport-entropy inequalities for $\mu \implies$ Concentration properties for μ.

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Specific example : the Ewens distribution on the symmetric group

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Specific example : the Ewens distribution on the symmetric group

μ^θ : the Ewens distribution of parameter $\theta > 0$ on the symmetric group S_n,

$$\mu^\theta(\sigma) = \frac{\theta^{|\sigma|}}{\theta^{(n)}}, \sigma \in S_n,$$

where

- $|\sigma|$ is the number of cycles in the cycle decomposition of σ,
- $\theta^{(n)}$ is the Pochhammer symbol defined by $\theta^{(n)} = \dfrac{\Gamma(\theta + n)}{\Gamma(\theta)}$,

$$\Gamma(\theta) = \int_0^{+\infty} s^{\theta-1} e^{-s} ds.$$

Result : The Chinese restaurant process. μ^θ is the law of the product of transpositions

$$(n, U_n)(n-1, U_{n-1}) \cdots (2, U_2),$$

where the U_i's are independent random variables with values in $\{1, \ldots, i\}$ and

$$\mathbb{P}(U_i = i) = \frac{\theta}{\theta + i - 1}, \quad \mathbb{P}(U_i = 1) = \cdots = \mathbb{P}(U_i = i-1) = \frac{1}{\theta + i - 1}.$$

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Particular case : $\theta = 1$,

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It follows that

$$\begin{aligned}
\widetilde{Q}\varphi(\sigma) &= \inf_{p\in\mathcal{P}(S_n)}\left\{\int \varphi(\tau)dp(\tau) + \frac{1}{20}\sum_{i=1}^n\left(\int 1_{\sigma(i)\neq\tau(i)}dp(\tau)\right)^2\right\}\\
&\geqslant \varphi(\sigma) - \sup_p \sum_{i=1}^n\left[\alpha_i(\sigma)\int \mathbb{1}_{\sigma(i)\neq\tau(i)}dp(\tau) - \frac{1}{20}\left(\int 1_{\sigma(i)\neq\tau(i)}dp(\tau)\right)^2\right]\\
&\geqslant \varphi(\sigma) - \sum_{i=1}^n \sup_{l\geqslant 0}\left\{\alpha_i(\sigma)l - \frac{l^2}{20}\right\}\\
&= \varphi(\sigma) - 5\sum_{i=1}^n \alpha_i^2(\sigma)\\
&= \varphi(\sigma) - 5|\alpha(\sigma)|_2^2,
\end{aligned}$$

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- $\varphi(\sigma) = \sup_{t \in \mathcal{F}} \sum_{k=1}^n a^t_{k,\sigma(k)}$, with $0 \leqslant a^t_{k,\sigma(k)} \leqslant M$, then for all $t \geqslant 0$,

$$\mu^\theta(\varphi \leqslant \mu^\theta(\varphi) - t) \leqslant \exp\left(-\frac{t^2}{20 \mu^\theta(\psi)}\right),$$

  and

$$\mu^\theta(\varphi \geqslant \mu^\theta(\varphi) + t) \leqslant \exp\left(-\frac{t^2}{20(\mu^\theta(\psi) + Mt)}\right),$$

  where $\psi(\sigma) = \sup_{t \in \mathcal{F}} \sum_{k=1}^n (a^t_{k,\sigma(k)})^2 \leqslant M\varphi(\sigma)$.

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$\mathcal{X}$: the state space of an homogenous Markov process,

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$\mathcal{X}$: the state space of an homogenous Markov process, $\mathcal{X}$ is discrete.

L : the infinitesimal generator.
$L^\gamma = \gamma L, \ \gamma > 0$ is the temperature.

m : a reversible mesure on $\mathcal{X}$, $m(x) > 0$, $\forall x \in \mathcal{X}$,

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$$R_{0,1}^\gamma(x,y) = m(x)P_\gamma(x,y), \qquad R_{0,1}^\gamma = m \oslash P_\gamma.$$

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- The dynamic Schrödinger problem associated to R^γ is

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Theorem : [see C. Léonard 2013]

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Theorem : [see C. Léonard 2013]

1. The dynamic and static Schrödinger problems have same minimum value,

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Theorem : [see C. Léonard 2013]

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$$T_S^\gamma(\mu_0, \mu_1) = \inf_{\pi \in \Pi(\mu_0,\mu_1)} H(\pi|R^\gamma_{0,1}).$$

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Theorem : [see C. Léonard 2013]

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Theorem : [see C. Léonard 2013]

1. The dynamic and static Schrödinger problems have same minimum value,
$$T_S^\gamma(\mu_0, \mu_1) = \inf_{\pi \in \Pi(\mu_0, \mu_1)} H(\pi|R^\gamma_{0,1}).$$
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The Schrödinger minimization problem

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Curvature in discrete setting

Question : Is there a “good” notion of curvature in discrete setting from which we can recover

- transport-entropy inequalities,
- Poincaré inequalities ,
- modified log-Sobolev inequalities, hypercontractivity,
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Several notions of curvature have been proposed on discrete spaces to extend the lower bound on Ricci-curvature in Riemannian geometry.

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Question : Is there a “good” notion of curvature in discrete setting from which we can recover

- transport-entropy inequalities,
- Poincaré inequalities ,
- modified log-Sobolev inequalities, hypercontractivity,
- Prékopa-Leindler types of inequalities,
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- The Bakry-Emery curvature condition (1985) - Γ_2-calculus,

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We will focus on the approach by C. Leonard in discrete, following the recent approach by G. Conforti (2018) in continuous spaces when L is a diffusion generator $Lf = \frac{1}{2}(\Delta f - \nabla U \cdot \nabla)$.

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