Criteria for entropic curvature on discrete spaces through the lens of an example: the hypercube

New Monge Problems and Applications - Martin Rapaport, LAMA, Université Gustave Eiffel

Based on joint work with Paul-Marie Samson *Criteria for entropic curvature on discrete spaces* (Arxiv)

Outline

Definition of Ricci curvature via *Optimal Transport*

Theorem (Otto-Villani '00,Cordero-McCann-Sumuckenslager '01,von Ranesse-Sturm '05)

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For this purpose, let us briefly introduce the *Discrete setting* and *Schrödinger bridges*.

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Some definitions, basic assumptions and dynamics

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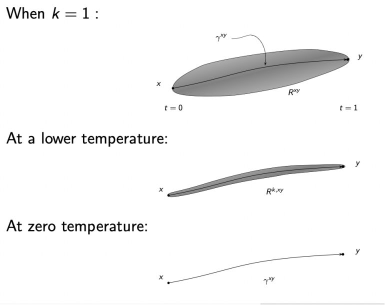

Figure: Image by C. Léonard illustrating the cooling procedure to obtain a Schrödinger bridge

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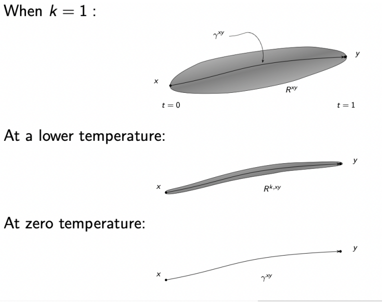

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Preliminaries to the Main Theorem

Local information: the function $K(z, S_2(z))$

For $k = 1$ or $k = 2$ and $z \in \mathcal{X}$ let *the combinatorial sphere* S_k:
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Discrete functional inequalities such as discrete modified logarithmic Sobolev inequality, Cheeger or discrete Poincaré type of inequalities can be derived.

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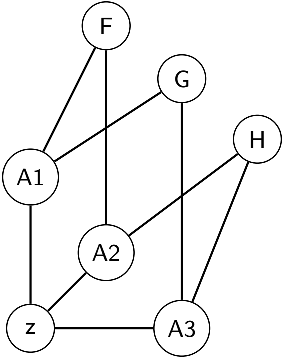

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In general for $\mathcal{Q}_n$, $K_0\big(z, S_2(z)\big) = 1 - \frac{1}{n}$ thus $\kappa = -2\log(1-\frac{1}{n})$; $\kappa_1 = \frac{4}{n}$.

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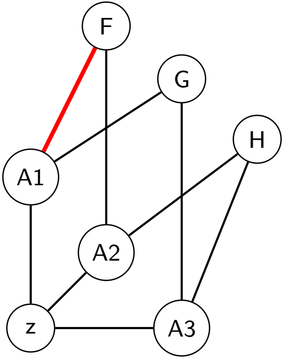

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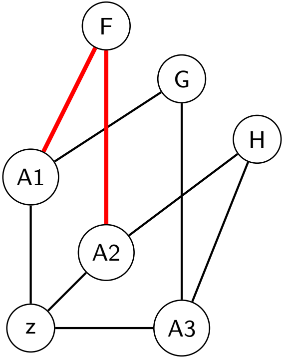

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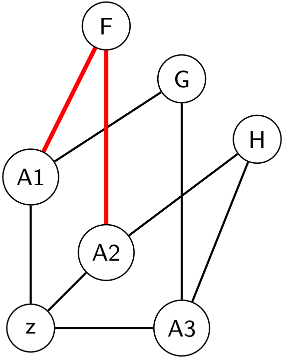

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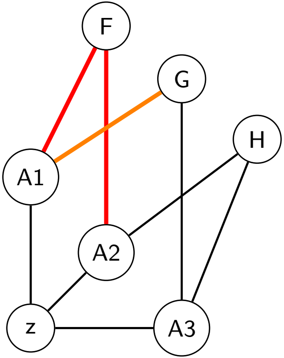

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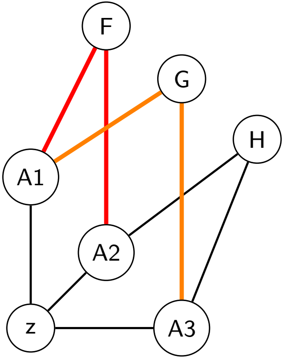

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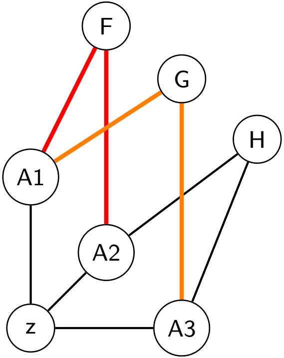

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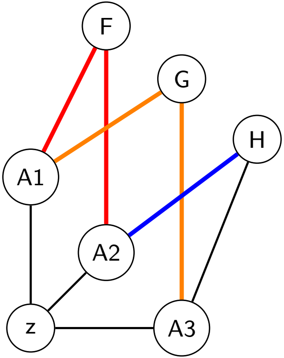

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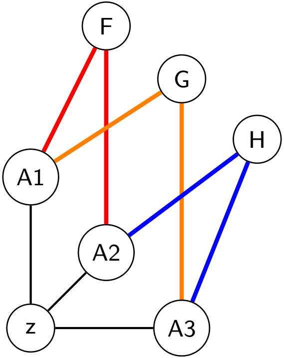

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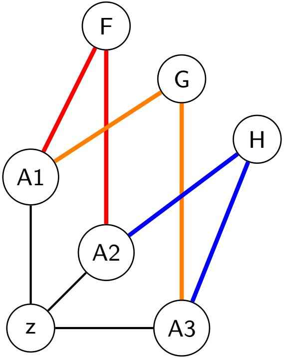

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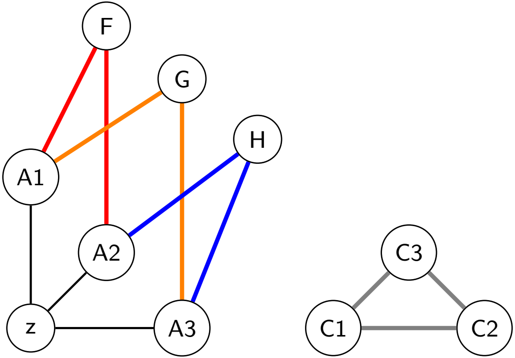

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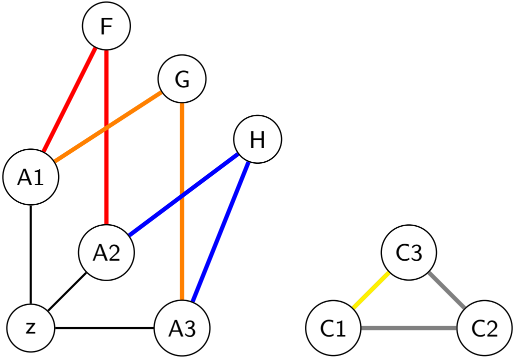

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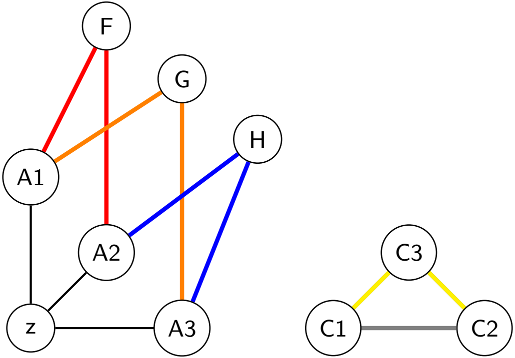

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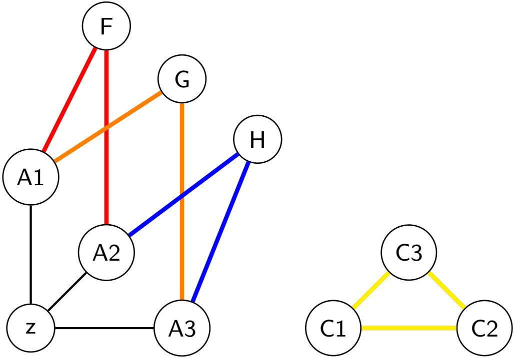

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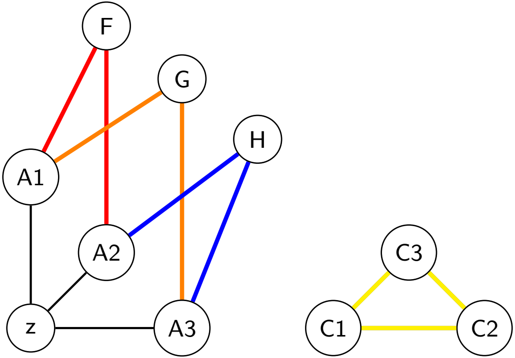

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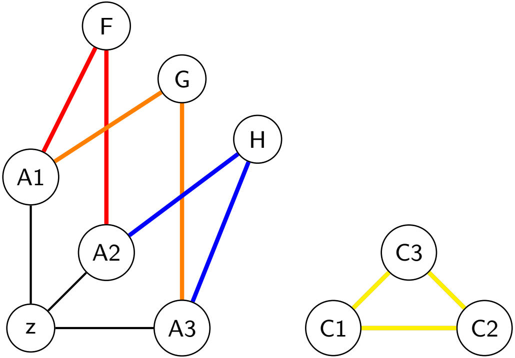

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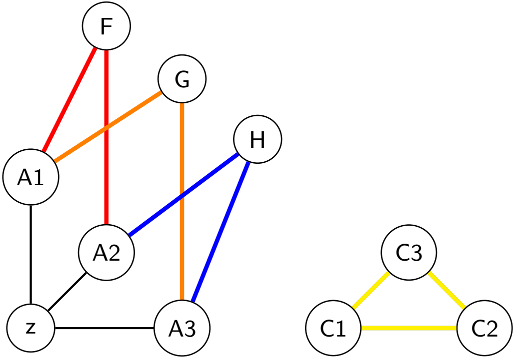

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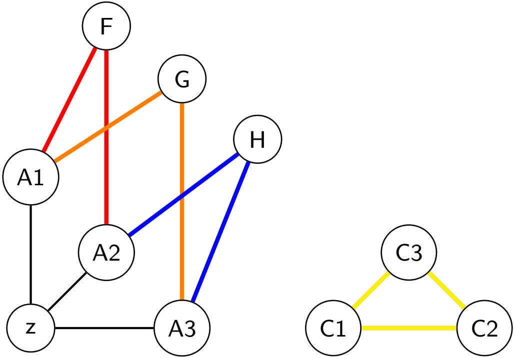

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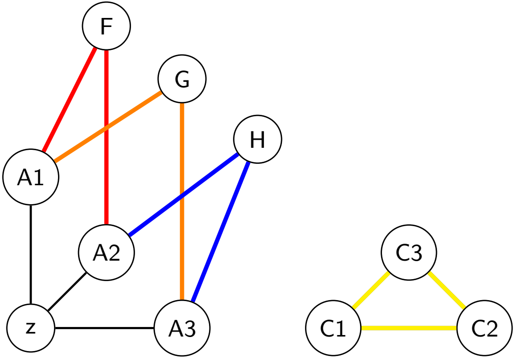

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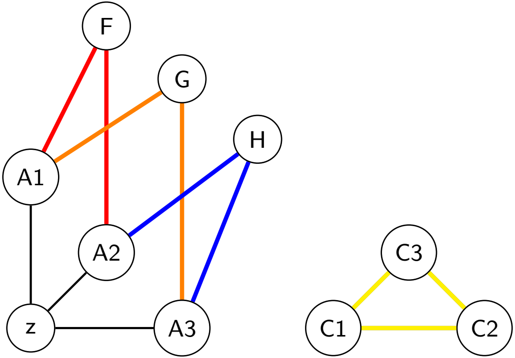

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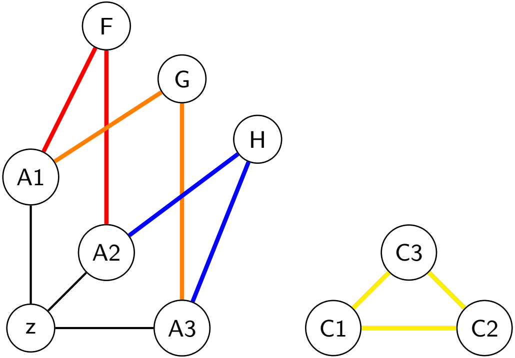

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